The Gerber Statistic: a Robust Co-movement Measure for Portfolio Optimization

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Key Message I

- We introduce the "Gerber statistic", a robust co-movement measure for covariance matrix estimation for the purpose of portfolio construction. The namesake of this statistic is the article's first author, Sander Gerber (CEO, Hudson Bay Capital); a precise formulation of the Gerber Statistic would later be achieved in collaboration with Harry Markowitz.
- The key idea of the Gerber statistic is to (i) recognize co-movement between series when the movements are substantial and (ii) to be insensitive to small co-movements that may be noise.

Key Message II

Using a well-diversified portfolio of nine assets over a 30-year period (January 1990-December 2020), we find that the Gerber statistic's returns dominate those of competing methods. We shall empirically show that the Gerber statistic **outperforms** historical correlation (HC) and the Ledoit-Wolf shrinkage (Ledoit and Wolf, 2004) on the key metrics of interest to any investor:

- Cumulative return.
- Average geometric return.
- Sharpe ratio.

Motivation: Modern Portfolio Theory

In modern portfolio theory (Markowitz 1952, 1959), portfolio construction relies on the availability of the matrix of covariances between securities' returns. Often the sample covariance matrix is used as an estimate for the actual covariance matrix (Jobson and Korkie, 1980).

A Key Problem with Sample Covariance

A key problem with the sample covariance matrix estimator (as well as many other covariance matrix estimators) is that it employs product-moment–based estimates that are inherently *not* robust. This is particularly troublesome if the underlying distribution of returns contains extreme measurements or outliers. Robust estimators, based on the pioneering work of Tukey (1960), Hampel (1968, 1974), and Huber (1977), have indeed largely worked to overcome this problem.

Do Existing "Robust" Estimators Work for Real Financial Data?

In short, we believe the answer is "NO". Some key reasons are:

- Financial time series have characteristics that make even standard robust techniques *unsuitable*. This is because financial time series are particularly noisy, and this noise can be easily misinterpreted as information.
- A consequence of this is that correlation matrix estimates (even those constructed with robust techniques) often have non-zero entries corresponding to series that in fact have no "meaningful" correlation.

The Gerber Statistic

In this talk, we shall solve some of the aforementioned problems by introducing the Gerber Statistic (GS), a robust *co-movement* measure that

- Ignores fluctuations below a certain threshold.
- Simultaneously limits the effects of extreme movements.

And, as we mentioned in the "Key Findings" section, we reemphasize that the Gerber Statistic is designed to:

- Recognize co-movement between series when the movements are *substantial*.
- To be insensitive to *small* co-movements that may be due to noise alone.

Performance Benchmarks for the Gerber Statistic

We confine our analysis to the mean-variance optimization (MVO) framework of Markowitz (1952, 1959). We will compare the performance of the Gerber Statistic (GS) to:

- The sample covariance matrix (also referred to as the historical covariance (HC) matrix, or simply "historical covariance" or "historical correlation").
- The shrinkage estimator of Ledoit and Wolf (2004), which shrinks the sample covariance matrix towards a structural estimator.

In contrast to the shrinkage estimator of Ledoit and Wolf (2004), the Gerber Statistic does not rely on the sample covariance as input.

Formulation of the Gerber Statistic

The goal of this part of the talk is to introduce (i) Gerber statistic and the (ii) Gerber correlation matrix, which is then converted to a (iii) Gerber covariance matrix that is inputted into the mean-variance portfolio optimizer.

Notation:

- We consider $k = 1, \ldots, K$ securities and $t = 1, \ldots, T$ time periods.
- Let r_{tk} be the return of security k at time t. For each pair (i, j) of assets for each time t, we convert the return observation of pair (r_{ti}, r_{tj}) to a joint observation $m_{ij}(t)$.

Assignment Mechanism

We convert the return observation of pair (r_{ti}, r_{tj}) to a joint observation $m_{ij}(t)$ given by the assignment mechanism below

$$m_{ij}(t) = \begin{cases} +1 \text{ if } r_{ti} \geq +H_i \text{ and } r_{tj} \geq +H_j, \\ +1 \text{ if } r_{ti} \leq -H_i \text{ and } r_{tj} \leq -H_j, \\ -1 \text{ if } r_{ti} \geq +H_i \text{ and } r_{tj} \leq -H_j, \\ -1 \text{ if } r_{ti} \leq -H_i \text{ and } r_{tj} \geq +H_j, \\ \quad 0 \text{ otherwise}, \end{cases} \tag{1}$$

where H_k is a threshold for security k and is calculated as

$$H_k = cs_k, \tag{2}$$

where c is some fraction (typically set to $1/2$, but may also be increased to $7/10$ or $9/10$) and s_k is the sample standard deviation of the return of security k.

A Conceptual View

- The joint observation $m_{ij}(t)$ is set to $+1$ if the series i and j simultaneously pierce their thresholds in the same direction at time t.
- The joint observation $m_{ij}(t)$ is set to -1 if the series i and j simultaneously pierce their thresholds in opposite directions at time t.
- The joint observation $m_{ij}(t)$ is set to 0 if at least one of the series does not pierce its threshold at time t.

A "First Crack" at a Formulation for the Gerber Statistic

We now consider the following statistic for a pair of assets

$$g_{ij} = \frac{\sum_{t=1}^{T} m_{ij}(t)}{\sum_{t=1}^{T} \left| m_{ij}(t) \right|}. \tag{3}$$

Note that in both the numerator and denominator, a joint observation must exceed its threshold before to be "counted" (that is, it is assigned a value of m_{ij} that is either $+1$ or -1). It is also important to note that since (3) relies on the counts of simultaneous piercings of thresholds, and not on the extent to which the thresholds are pierced, it is

- Insensitive to extreme movements that distort product moment-based measures.
- Insensitive to small movements that may simply be noise.

Covariance Matrices

Fact: A covariance matrix of securities' returns must be positive semidefinite (this ensures that variances are greater than or equal to zero, as they should be).

However, when working with real data, we found that the covariance matrix corresponding to the statistic in (3) was often not positive semidefinite (it is also straightforward to show this analytically). This led us to develop an alternative form of the statistic in (3) which gives rise to a positive semidefinite covariance matrix. It is this alternative form of (3) that we officially call the "Gerber Statistic".

A Reworking of the Statistic: Concordant and Discordant Pairs

Borrowing from the language of Kendall's Tau (Kendall, 1938), we shall refer to a pair for which both components pierce their thresholds while moving in the same direction as a *concordant* pair, and to one whose components pierce their thresholds while moving in opposite directions as a *discordant* pair. Letting n_{ij}^c be the number of concordant pairs for series i and j, and letting n_{ij}^d be the number of discordant pairs, equation (3) is immediately equivalent to

$$g_{ij} = \frac{n_{ij}^c - n_{ij}^d}{n_{ij}^c + n_{ij}^d}. \tag{4}$$

A Generalization of Kendall's Tau

One may easily see that the statistic in (4) is identical to Kendall's Tau (Kendall, 1938)

$$\tau = \frac{(\text{number of concordant pairs}) - (\text{number of discordant pairs})}{(\text{total number of pairs})} \tag{5}$$

if the threshold H_k is set to zero for all k. Thus, the Gerber statistic may be seen as a generalization of Kendall's Tau for portfolio management!

A Graphical Representation of Securities

We need to do a bit more work to derive a statistic which gives rise to a positive semidefinite matrix. We will be greatly aided in this task by considering the following graphical representation for the relationship between two securities in the figure below.

Notation

- The letter U represents the case in which a security's return lies above the upper threshold (i.e., is up).
- The letter N represents the case in which a security's return lies between the upper and lower thresholds (i.e., is neutral).
- The letter D represents the case in which a security's return lies below the lower threshold (i.e., is down).

A Graphical Representation

In the figure below, the rows represent categorizations of security i and the columns represent categorizations of security j.

A concrete example is now timely. At time t, the return of security i is above the upper threshold, this observation lies in the top row. If, at the same time t, the return of security j lies between the two thresholds, then this observation lies in the middle column. Therefore, for this example, this observation would lie in the UN region.

Figure: A graphical relationship between two securities.

$$\begin{matrix} UD & UN & UU \\ ND & NN & NU \\ DD & DN & DU \end{matrix}$$

An Equivalent Expression to (4)

Over the history, $t = 1, \ldots, T$, there will be observations scattered over the nine regions. Let n_{ij}^{pq} be the number of observations for which the returns of securities i and j lie in regions p and q, respectively, for $p, q \in \{U, N, D\}$. With this notation in hand, the following is an equivalent expression to the statistic presented in (4)

$$g_{ij} = \frac{n_{ij}^{UU} + n_{ij}^{DD} - n_{ij}^{UD} - n_{ij}^{DU}}{n_{ij}^{UU} + n_{ij}^{DD} + n_{ij}^{UD} + n_{ij}^{DU}}. \tag{6}$$

And Finally, the Gerber Statistic!

As previously noted, we must alter denominator in (4) to obtain a Gerber matrix which yields a corresponding covariance matrix in positive semidefinite form. Our alternative choice, which we call the "Gerber statistic," is

$$g_{ij} = \frac{n_{ij}^{UU} + n_{ij}^{DD} - n_{ij}^{UD} - n_{ij}^{DU}}{T - n_{ij}^{NN}}. \tag{7}$$

The Gerber Matrix

The Gerber matrix $\boldsymbol{G}$ is the matrix that contains the the Gerber statistic g_{ij} in its i-th row and j-th column. It is straightforward to analytically show that the covariance matrix obtained from the Gerber matrix $\boldsymbol{G}$ is positive semidefinite. We now have in hand the Gerber covariance matrix to be used in MVO.

$\Sigma_{GS} = \text{diag}(\boldsymbol{\sigma})\boldsymbol{G}\text{diag}(\boldsymbol{\sigma})$, where $\boldsymbol{G}$ is the Gerber matrix obtained from the Gerber statistic in (7) and $\boldsymbol{\sigma}$ is a $N \times 1$ vector of sample standard deviations of the historical asset returns.

Calculation of the Gerber Statistic

We provide a brief example to illustrate how the Gerber statistic is calculated between a given pair of assets.

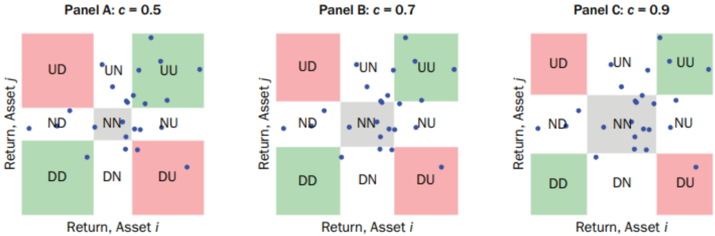
In the figure below, we compute 24 pairwise monthly returns between the assets S&P 500 (SPX) and Gold (XAU) for the period from January 2019 to December 2020. Recalling that the threshold of H_k as defined above in (2) as

$$H_k = cs_k,$$

where c is some fraction (typically set to 1/2, but may also be increased to 7/10 or 9/10) and s_k is the sample standard deviation of the return of security k. We consider three different values of c: $c = .5$, $c = .7$, and $c = .9$.

A Picture is Worth a Thousand Words!

Illustration of Pairwise Returns for Evaluating the Gerber Statistics given $c = 0.5$, $c = 0.7$, and $c = 0.9$



NOTES: Each pairwise monthly return appears as a blue dot. The points in the green zone correspond to the concordant pairs, whereas the points falling in the red zone are discordant pairs. The return series of assets i and j are transformed to {–1, 0, 1} using upper and lower thresholds calculated by Equation 2.

A Picture is Worth a Thousand Words!

Recall we defined the "Gerber statistic" above as

$$g_{ij} = \frac{n_{ij}^{UU} + n_{ij}^{DD} - n_{ij}^{UD} - n_{ij}^{DU}}{T - n_{ij}^{NN}}.$$

The key intuition for our choice of the Gerber statistic's denominator in (7) comes from the following observation: as c becomes larger, more data points are included in the region NN. This leads to the statistic becoming more robust and less sensitive to noise in the data. We refer to this artifact of the Gerber statistic as "stripping noise" from the data.

A Calculation for $c = .7$

We now calculate the Gerber statistic by counting the points falling into each region. The results for the case $c = .7$ is given below. In the case $c = 0.7$, the counts for nine regions are $n_{ij}^{UD} = 0$, $n_{ij}^{UN} = 5$, $n_{ij}^{UU} = 4$, $n_{ij}^{ND} = 3$, $n_{ij}^{NN} = 6$, $n_{ij}^{NU} = 2$, $n_{ij}^{DD} = 0$, $n_{ij}^{DN} = 3$ and $n_{ij}^{DU} = 1$. Employing the formula for the Gerber statistic in (7), we obtain that

$$g_{ij} = \frac{4 + 0 - 0 - 1}{24 - 6} = \frac{1}{6} \approx 0.166.$$

Unsurprisingly, this differs from the standard Pearson correlation of .22.

How Does the Gerber Statistic Differ from Pearson Correlation?

In concordance with Pearson's correlation coefficient (and Kendall's Tau coefficient), the value of the Gerber statistic is also always contained in the interval $[-1, 1]$. However, there are key conceptual differences between the Gerber statistic and the Pearson correlation coefficient. They are as follows:

Differences

- The Pearson correlation coefficient inputs the sample covariance of assets i and j and the sample standard deviation of assets i and j (and therefore the sample means of assets i and j). By definition, the sample covariance, the sample mean, and the sample standard deviation are calculated over *all* data points, regardless of whether the points correspond to meaningful co-movement or to pure noise. This causes the Pearson correlation to be highly sensitive to small co-movements that may be due to noise alone.
- In contrast, the numerator of the Gerber statistic in (7) only includes the *subset* of the dataset containing the points corresponding to meaningful co-movement; that is, the Gerber statistic *strips away* "noisy" data. We see this to be the key reason that the Gerber statistic is a more robust co-movement measure than the standard Pearson correlation.

Differences (continued)

Unlike the Pearson correlation coefficient, the Gerber statistic is "almost" entirely non-parametric. Indeed, we could achieve an entirely moment-free framework for the Gerber statistic by replacing s_k in equation (2) with a more robust measure of standard deviation. We shall explore candidates for this measure in future work.

Comparison

We will compare the performance of the Gerber covariance matrix in Markowitz's MVO to the following benchmarks:

- The sample historical covariance matrix $\boldsymbol{\Sigma}_{\mathsf{HC}}$.
- The shrinkage method of Ledoit and Wolf (2004) $\boldsymbol{\Sigma}_{\mathsf{SM}}$

$$\boldsymbol{\Sigma}_{\mathsf{SM}} = \delta \boldsymbol{\Sigma}_F + (1 - \delta)\, \boldsymbol{\Sigma}_{\mathsf{HC}}, \tag{8}$$

where δ is a shrinkage constant between 0 and 1. In choosing δ, Ledoit and Wolf (2004) propose finding the shrinkage parameter by minimizing the Frobenius norm between the asymptotically true covariance matrix $\boldsymbol{\Sigma}$ and the shrinkage estimator $\boldsymbol{\Sigma}_{\mathsf{SM}}$. In computing $\boldsymbol{\Sigma}_F$, Ledoit and Wolf (2004) suggest a constant correlation model, i.e. the average sample correlation of all pairs for the nondiagonal elements of the sample correlation matrix.

Backtesting Procedure

The portfolio optimization framework we shall consider is that of MVO with a transaction fee of 10 basis points of traded volume. The following backtesting procedure is employed to benchmark performance among different covariance estimators for portfolio optimization.

Backtesting Procedure

- Beginning January 1990, at the beginning of each month, the monthly returns of the current list of assets from a lookback window of 24 months are utilized to estimate: (i) The expected return vector and (ii) The covariance matrix. Each of these are then inputted into the mean-variance optimizer.
- All portfolios are rebalanced on a *monthly* basis. We repeat this rebalancing process by moving the in-sample period one month forward and computing the updated efficient portfolio for the next month.
- Note that because two years worth of monthly returns are required to initialize the first portfolio, our performance evaluation ranges from the period January 1990 to December 2020.

Empirical Work

We now commence the empirical study of the performance of the Gerber statistic in comparison to our two benchmarks: historical covariance and the shrinkage estimator of Ledoit and Wolf (2004).

Dataset

The dataset we consider is a well diversified collection of nine assets over the time period January 1988 to December 2020, as follows:

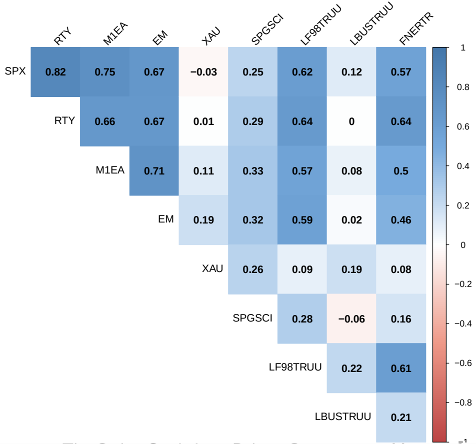
1. S&P 500 index (U.S. large-cap stocks; Ticker SPX)
2. Russell 2000 index (U.S. small-cap stocks; Ticker RTY)
3. MSCI EAFE index (captures large and mid cap equities across twenty-one developed countries excluding U.S. and Canada)
4. MSCI Emerging Markets index: Ticker MXEF)
5. Bloomberg Barclays U.S. Aggregate Bond index: Ticker LBUSTRUU)
6. Bloomberg Barclays U.S. Corporate High Yield Bond index; Ticker LF98TRUU
7. Real estate FTSE NAREIT all equity REITS index; Ticker FNERTR
8. Gold; Ticker XAU
9. S&P GSCI Commodity index; Ticker SPGSCI

Diversification of Assets

Since the nine assets above are well-diversified, we do not expect to observe a “strong” pairwise correlation structure between the assets. This is confirmed by the figure below, which displays a correlation matrix of the total return series from January 1990 to December 2020 for the nine assets.

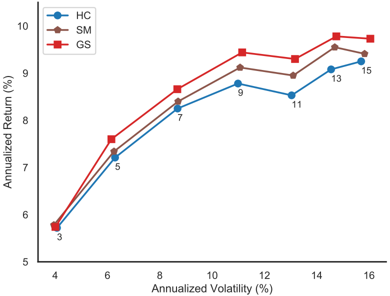
A Correlation Matrix

Figure: Heat map of the correlation matrix (given the total return series from January 1990 to December 2020) for the nine assets.



Results I: Gerber Statistic: $c = .5$

Figure: The realized performance in terms of annualized return and annualized volatility of portfolios (the ex-post efficient frontiers) with different risk target levels from 3% to 15%, with an increment of 2%. The blue frontier illustrates the ex-post performance of HC-based portfolios, the brown the SM-based portfolios, and the red the GS-based portfolios.



Results I: Gerber Statistic: $c = .5$

1. For all risk target levels, the Gerber statistic offers a more favorable risk-return profile than HC. With the exception of the ultra-conservative risk target level of 3%, the Gerber statistic offers a more favorable risk-return profile than SM.
2. For all risk targets, the Gerber statistic yields higher cumulative returns than HC. With the exception of the very conservative risk target level of 3%, the Gerber statistic yields higher cumulative returns than SM.
3. With similar values of portfolio turnover, skewness, and kurtosis as both the HC and SM portfolios, the Gerber statistic posts higher geometric returns and higher Sharpe ratios than HC across all risk target levels.
4. With the exception of the very conservative risk target level of 3%, the Gerber statistic yields higher geometric returns and higher Sharpe ratios than SM across all other risk target levels.

Performance Comparisons: Average Annualized Geometric Return

For some risk target levels, the average annualized geometric return of GS is more than 30 basis points higher than that of SM and more than 75 basis points higher than HC. The latter result is unsurprising given the limitations of HC, and so we instead focus on the advantages of GS over SM.

- For the 9% risk target level, the average annualized geometric return of GS is approximately 32 basis points higher than that of SM and its cumulative return is 10.16% higher than that for SM over the 1990–2020 period.
- For the 15% risk target level, the average annualized geometric return of GS is approximately 32 basis points higher than that for SM and its cumulative return is 10.32% higher than SM over the 1990–2020 period.

Cumulative Returns for $c = .5$

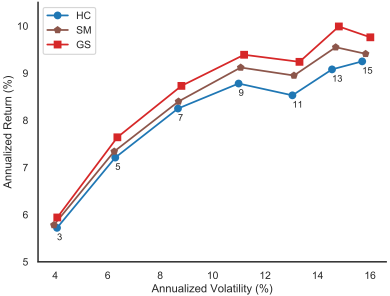
Account Dollar Value in December 2020 for HC-Based Portfolios, SM-Based Portfolios, and GS-Based Portfolios at Five Different Annual Risk Target Levels (3%, 6%, 9%, 12%, and 15%), given the Gerber Threshold $c = 0.5$

Method	HC	SM	GS
Ultra-Conservative (3%)	$561,276.27	$570,161.82	$564,972.97
Conservative (6%)	$1,020,099.74	$1,058,096.16	$1,138,042.25
Moderate (9%)	$1,356,911.18	$1,497,204.43	$1,639,089.77
Aggressive (12%)	$1,364,148.39	$1,584,491.30	$1,695,255.50
Ultra-Aggressive (15%)	$1,551,338.93	$1,622,590.70	$1,779,756.04

NOTE: The calculation assumes that $100,000 is invested in January 1990 and is left to grow according to portfolio weights determined by each covariance method and risk target level until December 2020.

Results II: Gerber Statistic: $c = .7$

Figure: The realized performance in terms of annualized return and annualized volatility of portfolios (the ex-post efficient frontiers) with different risk target levels from 3% to 15%, with an increment of 2%, given the Gerber threshold $c = 0.7$. The blue illustrates the ex-post performance of HC-based portfolios, the brown that of SM-based portfolios, and the red frontier corresponds to GS-based portfolios.



Results II: Gerber Statistic: $c = .7$

1. **For all risk target levels**, the Gerber statistic offers a more favorable risk-return profile than **both** HC and SM.
2. **For all risk target levels**, the Gerber statistic offers superior cumulative returns to **both** HC and SM.
3. **For all risk target levels**, the Gerber statistic gives higher geometric returns and Sharpe ratios to **both** HC and SM, and has similar values of portfolio turnover, skewness, and kurtosis to HC and SM.

Results II: Gerber Statistic: $c = .7$

For some risk target levels, the average annualized geometric return of GS is more than 40 basis points higher than that of SM and is more than 90 basis points higher than HC. The latter is unsurprising given the limitations of HC, and so we instead focus on the advantages of GS over SM.

- For the 12% risk target level, the average annualized geometric return of GS is approximately 41 basis points higher than that of SM and its cumulative return is 13.12% higher than that for SM over the 1990–2020 period.
- For the 15% risk target level, the average annualized geometric return of GS is approximately 35 basis points higher than that for SM and its cumulative return is 11.18% higher than than SM over the 1990–2020 period.

Cumulative Returns for $c = .7$

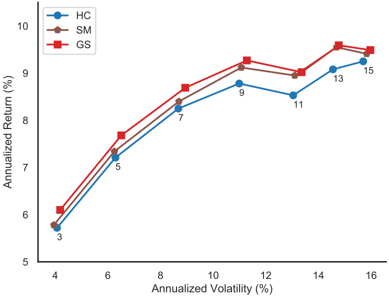
Dollar Value in December 2020 for HC-Based Portfolios, SM-Based Portfolios, and GS-Based Portfolios under Five Annual Risk Target Levels (3%, 6%, 9%, 12%, and 15%), with a Gerber Threshold c = 0.7

Method	HC	SM	GS
Ultra-Conservative (3%)	$561,276.27	$570,161.82	$599,043.19
Conservative (6%)	$1,020,099.74	$1,058,096.16	$1,151,575.55
Moderate (9%)	$1,356,911.18	$1,497,204.43	$1,617,126.27
Aggressive (12%)	$1,364,148.39	$1,584,491.30	$1,779,267.42
Ultra-Aggressive (15%)	$1,551,338.93	$1,622,590.70	$1,792,846.87

NOTE: The calculation assumes that $100,000 is invested in January 1990 and is left to grow until December 2020.

Results III: Gerber Statistic: $c = .9$

Figure: The realized performance in terms of annualized return and annualized volatility of portfolios (the ex-post efficient frontiers) with different risk target levels from 3% to 15%, with an increment of 2%. The blue frontier illustrates the ex-post performance of HC-based portfolios, the brown the SM-based portfolios, and the red frontier the GS-based portfolios.



Results III: Gerber Statistic: $c = .9$

- **For all risk target levels**, the Gerber statistic offers a more favorable risk-return profile than **both** HC and SM.
- **For all risk target levels**, the Gerber statistic offers superior cumulative returns to **both** HC and SM.
- **For all risk target levels**, the Gerber statistic posts higher geometric returns and Sharpe ratios to **both** SM and HC, and has similar values of portfolio turnover, skewness, and kurtosis to HC and SM.

Results III: Gerber Statistic: $c = .9$

- For the 3% and 6% risk target levels, the average annualized geometric return of GS is, respectively, approximately 32 and 35 basis points higher than those of SM.
- The cumulative returns are, respectively, 12.11% and 11.67% higher than those for SM over the 1990–2020 period.
- We also note that for the 6% risk target level, the average annualized geometric return of GS is more than 48 basis points higher than that of HC.

Cumulative Returns for $c = .9$

Account Dollar Value in December 2020 for HC-Based Portfolios, SM-Based Portfolios, and GS-Based Portfolios at Five Different Annual Risk Target Levels (3%, 6%, 9%, 12%, and 15%), given the Gerber Threshold $c = 0.9$

Method	HC	SM	GS
Ultra-Conservative (3%)	$561,276.27	$570,161.82	$627,077.52
Conservative (6%)	$1,020,099.74	$1,058,096.16	$1,169,911.24
Moderate (9%)	$1,356,911.18	$1,497,204.43	$1,560,198.02
Aggressive (12%)	$1,364,148.39	$1,584,491.30	$1,607,572.21
Ultra-Aggressive (15%)	$1,551,338.93	$1,622,590.70	$1,660,628.94

NOTE: The calculation assumes that $100,000 is invested in January 1990 and is left to grow according to portfolio weights determined by each covariance method and risk target level until December 2020.

Conclusion

- We have introduced a co-movement measure called the Gerber statistic.
- The Gerber statistic is well-suited for assessing co-movement between financial time series because it is insensitive to extremely large co-movements that distort product-moment-based measures.
- The Gerber statistic is also insensitive to small movements that are likely to be noise.
- We have studied the performance of the Gerber statistic within the mean-variance portfolio optimization framework of Markowitz (1952, 1959).
- In *all 15* investment scenario considered, the Gerber statistic's performance is superior to that of historical covariance. In *14 of 15* investment scenario considered, the Gerber statistic dominates the shrinkage estimator on the key metrics of interest to any investor: cumulative return, average geometric return, and Sharpe ratio.

Conclusion

The Gerber statistic is easy to compute and is straightforward to implement in a mean-variance optimizer. Our hope is that it will become a welcome alternative to both historical covariance and to the shrinkage estimator of Ledoit and Wolf (2004).

Thank You

THANK YOU!

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Paper Available on Hudson Bay's Website (https://www.hudsonbaycapital.com/research_news) as well as on the website of *The Journal of Portfolio Management*.


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EXHIBIT 6

Performance Metrics for HC-, SM-, and GS-Based Portfolios at Five Different Risk Target Levels (3%, 6%, 9%, 12%, and 15%) for the Full Testing Period between January 1990 and December 2020

	Ultra-Conservative (3%)			Conservative (6%)			Moderate (9%)			Aggressive (12%)			Ultra-Aggressive (15%)		
Covariance Method	HC	SM	GS	HC	SM	GS	HC	SM	GS	HC	SM	GS	HC	SM	GS
Arithmetic Return (%)	5.83	5.90	5.85	8.10	8.22	8.46	9.41	9.77	10.09	9.69	10.32	10.50	10.54	10.78	11.17
Geometric Return (%)	5.72	5.78	5.74	7.78	7.91	8.16	8.78	9.12	9.44	8.79	9.32	9.56	9.25	9.41	9.73
Cumulative Return (%)	461.28	470.16	464.97	920.10	958.10	1,038.04	1,256.91	1,397.20	1,539.09	1,264.15	1,484.49	1,595.26	1,451.34	1,522.59	1,679.76
Annualized SD (%)	4.07	3.95	4.01	7.49	7.50	7.43	10.99	11.07	11.15	13.82	13.92	13.98	15.70	15.83	16.05
Annualized Skewness	–0.90	–0.90	–0.88	–0.93	–0.98	–0.99	–0.85	–0.88	–0.93	–0.93	–0.92	–0.93	–0.86	–0.86	–0.87
Annualized Kurtosis	5.21	5.37	5.31	5.53	5.85	6.07	4.97	5.11	5.60	5.27	5.30	5.56	5.34	5.42	5.55
Maximum Drawdown (%)	–10.03	–7.67	–7.73	–20.30	–19.10	–17.74	–28.83	–28.03	–26.58	–36.47	–35.22	–32.47	–41.76	–41.13	–43.01
Monthly 95% VaR (%)	–1.53	–1.52	–1.58	–2.85	–2.69	–2.68	–4.35	–4.29	–4.29	–5.99	–5.81	–5.84	–6.70	–6.59	–6.42
Sharpe Ratio	0.73	0.77	0.75	0.67	0.69	0.73	0.55	0.58	0.60	0.44	0.47	0.49	0.42	0.42	0.44
Annualized Turnover	1.80	1.39	1.58	2.78	2.53	2.49	3.69	3.40	3.37	4.45	4.27	4.19	4.46	4.33	4.23

NOTES: The three-month US T-bill rate was used as the risk-free rate. The transaction cost is modeled as 10 bps of the traded volume for each rebalancing event.

EXHIBIT 10

Performance Sensitivity Study of the GS-Based Portfolios for Thresholds $c = 0.5$, $c = 0.7$, and $c = 0.9$ at Five Different Risk Target Levels (3%, 6%, 9%, 12%, and 15%)

	Ultra-Conservative (3%)			Conservative (6%)			Moderate (9%)			Aggressive (12%)			Ultra-Aggressive (15%)		
GS Threshold c	0.50	0.70	0.90	0.50	0.70	0.90	0.50	0.70	0.90	0.50	0.70	0.90	0.50	0.70	0.90
Arithmetic Return (%)	5.85	6.06	6.22	8.46	8.52	8.59	10.09	10.05	9.94	10.50	10.68	10.34	11.17	11.17	10.90
Geometric Return (%)	5.74	5.94	6.10	8.16	8.20	8.26	9.44	9.39	9.27	9.56	9.73	9.37	9.73	9.76	9.49
Cumulative Return (%)	464.97	499.04	527.08	1,038.04	1,051.58	1,069.91	1,539.09	1,517.13	1,460.20	1,595.26	1,679.27	1,507.57	1,679.76	1,692.85	1,560.63
Annualized SD (%)	4.01	4.08	4.19	7.43	7.62	7.76	11.15	11.20	11.28	13.98	14.02	14.15	16.05	16.01	15.97
Annualized Skewness	–0.88	–1.06	–1.04	–0.99	–1.04	–0.98	–0.93	–0.90	–0.87	–0.93	–0.94	–0.98	–0.87	–0.89	–0.86
Annualized Kurtosis	5.31	6.26	6.13	6.07	6.30	5.84	5.60	5.42	5.12	5.56	5.58	5.66	5.55	5.59	5.38
Maximum Drawdown (%)	–7.73	–9.11	–9.67	–17.74	–19.08	–20.03	–26.58	–27.75	–28.12	–32.47	–33.94	–34.01	–43.01	–43.12	–40.94
Monthly 95% VaR (%)	–1.58	–1.50	–1.55	–2.68	–2.79	–2.82	–4.29	–4.37	–4.40	–5.84	–5.78	–5.71	–6.42	–6.49	–6.53
Sharpe Ratio	0.75	0.78	0.80	0.73	0.72	0.71	0.60	0.60	0.58	0.49	0.50	0.47	0.44	0.44	0.42
Annualized Turnover	1.58	1.49	1.51	2.49	2.55	2.58	3.37	3.43	3.44	4.19	4.19	4.26	4.23	4.21	4.32